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NUMERICAL SOLUTION OF THE SHALLOW WATER EQUATION BY USING SPREADSHEET

ABDULRAZAK HOMIDI H. ALMALIKI

Lecturer, Department of Civil Engineering, Faculty of Engineering, Taif University, Al Hawiyah, Saudi Arabia

ABSTRACT

Natural hazards occupy the essential and regional levels, hence, they are raised as a priority issues. The 2009 Saudi Arabia floods affected Jeddah, on the red sea (western) coast. As of January 3rd, 2010, 122 people are reported to have been killed. Roads were under a meter of water. Unfortunately, Lack of knowledge in water flow modeling contributes to prevent manage flood risks. This paper provided the spreadsheet model as solver to solve wave propagation due to sudden closure of the downstream gate or closed road. A Wave propagation flow is simulated by numerically solving the one-dimensional saint venant equations by using a second-order explicit finite-difference (McCormack) scheme. This method has been verified with using lux diffusive scheme. Spreadsheets as new solution technique is used to simulate unsteady flow in open channel which is a practical method. The results obtained by using the Saint Venant equations are compared to determine the depth at each elven sections which the maximum depth was 6.858437 m at time around 1050 second. It is found that, the Saint Venant equations give sufficiently accurate results for the maximum flow depth and the time to reach this value at a location downstream.

KEYWORDS: Saint Venant, Spreadsheets, McCormack Scheme

INTRODUCTION

Transient or unsteady flow in open channels can be called propagation of water waves. Mathematically, water waves propagation can be described by the Saint-Venant equations (assuming hydrostatic pressure distribution in the vertical direction) or by other equations such as simplified wave approximations, non-inertia wave, gravity wave and dynamic wave models. Since the 1970s different methods of numerical approaches have been developed in order to solve Saint-Venant equations or others, such as Liggett and Woolhiser (167), Wylie (1970) and Woolhiser (1977). The analysis of propagation of water waves is vital for maintenance, operation and emergency planning. Water wave propagation is studied by numerical solution of the Saint-Venent equations. This paper will focus on numerical solutions used for solving the Saint-Venent equations that describe and simulate water waves, water wave propagation and dam break simulation. The main goal of this paper is to understand water wave propagation.

Computational hydraulics is an important for water resource engineers. it is a tool of simulation water flow under certain condition to ease studying and visualize a certain case . In this paper McCormack scheme is used for simulating sudden closure of downstream gate and lux diffusive scheme in order to verification .recently some of published paper used spreadsheet (Excel) in solving complex 3 dimension numerical problem particularly in hydraulics field for instance HALIL KARAHAN, M. TAMER AYVAZ utilize spread cheat (excel) in Groundwater Modeling .spread cheat (excel) flexible and workable to use which is showing the result numerically and graphically. All numerical methods are subjected to stability constraints which restrict the values allowed for the time step for a given grid [1,2]. For explicit schemes, this can result computationally long .Better explicit schemes or implicit schemes overcome this disadvantage but with more complexity [2].

The saint venant equations or dynamic equation appear in many forms in the literature and can either be written as a set of integral or differential equations [4], the Saint Venant equations are a special case of the Navier-Stokes equations. In case of one dimension unsteady flow, momentum equation or energy equation and continuity equation(PDE) must be solved which are having at least 2 unknown variable the following method are used for solving them .

- Characteristic method
- Kinematic wave simulation
- Diffusive wave simulation
- Full dynamic simulation
- Finite deference
- Finite element
- Finite volume
- Finite boundary

PROBLEM STATEMENT

The propagation of water waves might cause dam failure and can cause significant damage, particularly when dams are located in a residential area, such as is the case in Jeddah. Water wave propagation is not completely understood, and from this point of view it is important to induce researchers to study this phenomenon.

METHODOLOGY

Seeking the scientific approach of using numerical solution. Deductive and inductive is adapted in this research

OBJECTIVE

This paper aims to improve the level of understanding of the propagation of water waves in open channels. The main approach of this paper will be numerical solutions used for solving the Saint-Venant equations that describe and simulate water waves, particularly, transient condition which are produced by the sudden closure of a downstream gate or closed road. Using spread cheat (excel) to simulate water waves.

FUNDAMENTAL EQUATION OF UNSTEADY FLOW

The flow of the water can be described mathematically based on three fundamental laws of physics as follows:

- Conservation of matter (or mass).
- Conservation of energy , (Bernoulli equation)
- Conservation of momentum.

The concepts is described briefly and given below.

GOVERNING EQUATIONS

Continuity equation and momentum or energy equation are the common governing equations which together form the differential version of the saint venent equations see equations. As our case assumption that the system has no energy losses, the covering equations are Continuity equation and momentum equations. By solving them we will obtain the variable in term of each other.

Momentum or Dynamic Equation

By applying Newton's 2nd law to our elemental length of channel we have f=mg. In case of unsteady flow here is the equation:

$$\frac{\partial Q}{\partial t} + \frac{\partial QV}{\partial x} + gA\frac{\partial y}{\partial x} = gA\left(S_0 - S_f\right) \quad (1)$$

Substitution of Q = AV into Eq. 1, equation leads to:

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} + g\frac{\partial y}{\partial x} = g\left(S_0 - S_f\right)$$

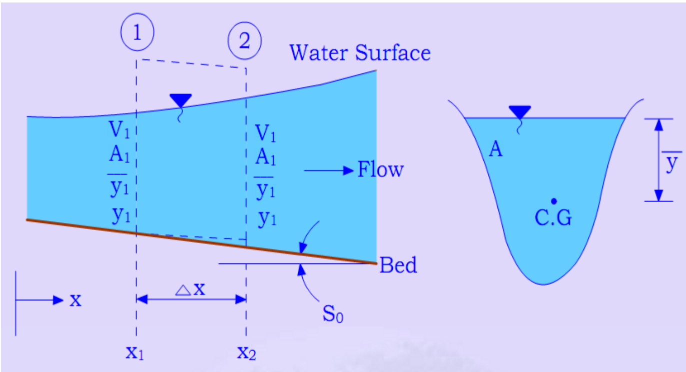


Figure 1: Definition Sketch for Derivation of St. Venant Equations

Continuity Equation

Unsteady flow can be written depends on mass (matter) conservation as flows:

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0$$

Substitution of Q = AV into equation, Continuity Equation leads to be:

$$\frac{\partial A}{\partial t} + \frac{\partial AV}{\partial x} = 0$$

One dimensional continuity equation can be written as (Chaudhry 2002):

$$\frac{\partial y}{\partial t} + \left(\frac{A}{T}\right)\frac{\partial V}{\partial x} + V\frac{\partial y}{\partial x} = 0 \quad (2)$$

Equation for celerity of a small gravity wave in a trapezoidal channel can be written as:

C = √ g*T/A

A FIRST NUMERICAL METHOD OF SOLUTION

Eqs. (1) and (2) are nonlinear analytical solutions which are both partial differential equations, a finite-difference method is used to solve these two governing equations. Therefore, a numerical approach is used here for their solution. It is necessary to use third or higher-order accurate numerical methods to solve these equations (Abbott 1979). In this paper, the method developed by McCormack and lux (1976) (Chaudhry 2003) are used for solving the saint venet equations for studying the wave of downstream dam gate sudden closure.

Two well-known numerical methods will be used in this case although they are both approximations. One is McCormack sachem and the anther is lux diffusive sachem which is both based on using the ideas of where information coming from which itself determined by the characteristics see figure [2].

Consider the arrangement of grid shown in Figure 3, in which the nodes are spaced at regular Δx intervals along the direction of the x axis and at Δt intervals along the t axis [2].

First, an intermediate flow field (v velocity ,h water depth) is computed by a predictor-corrector procedure see appendix , The partial derivatives of the dependent variables can be written as:

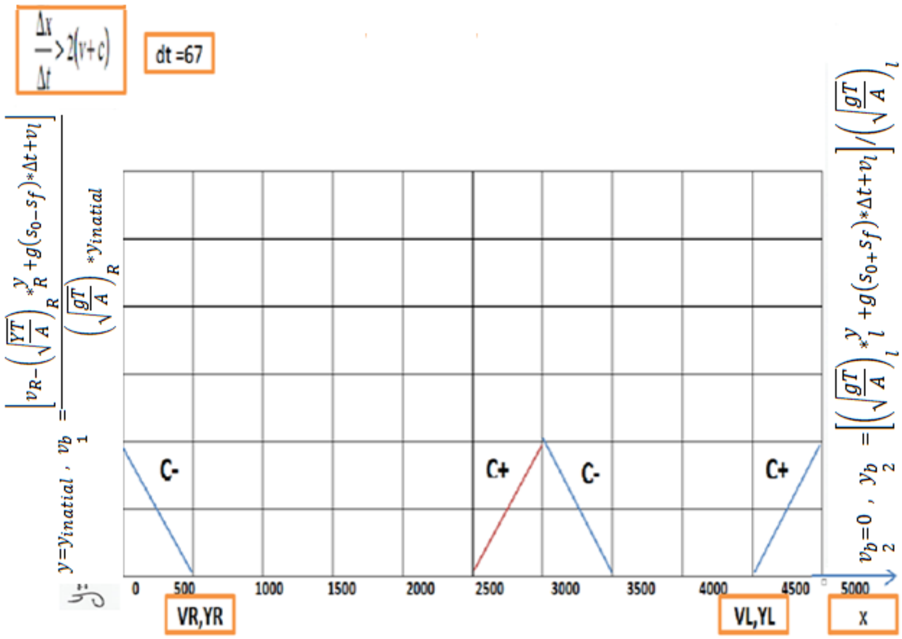


Figure 2: Staggered Grid

Left boundary conditions $v_{b_2}=0$, $y_{b_2} = \left[\left(\sqrt{\frac{gT}{A}}\right)_l * y_l + g(s_{0+}s_f)*\Delta t + v_l\right] / \left(\sqrt{\frac{gT}{A}}\right)_l$

Right boundary conditions

$$y=y_{inatial} \text{ , } v_{b_1} = \frac{\left[v_R-\left(\sqrt{\frac{YT}{A}}\right)_R * y_R + g(s_0-s_f)*\Delta t+v_l\right]}{\left(\sqrt{\frac{gT}{A}}\right)_R * y_{inatial}}$$

Where , $R_R=\frac{A_R}{T_R}$, $s_f=\frac{[n^2 v^2]}{\left(R^{\frac{4}{3}}\right)_R}$, $A_R=[\left(B+\frac{(B+2y_R*S)}{2}\right]*y_R$

The left and right boundry condtions are shown on the figure 2.interior value of v, y for the new time step can be written as predictor and then can be corrected to be called predictor , after that , by taking the average between the corrector and predictor we get the target value , the posture as flows:

Predictor

Substitution of these expressions for the partial derivatives and simplification and predictor can be written as follows:

$$\mathbf{U}_i^* = \mathbf{U}_i^k - \frac{\Delta t}{\Delta x}\left(\mathbf{F}_i^k - \mathbf{F}_{i-1}^k\right) - \mathbf{S}_i^k \Delta t$$

$$y_i^* = y_i^k - \frac{\Delta t}{\Delta x} D_i^* (V_i^k - V_{i-1}^k) - \frac{\Delta t}{\Delta x} V_i^k (y_i^k - y_{i-1}^k)$$
$$V_i^* = V_i^k - V_i^k \frac{\Delta t}{\Delta x} (V_i^k - V_{i-1}^k) - g \frac{\Delta t}{\Delta x} (y_i^k - y_{i-1}^k) - g \Delta t \left(S_o - S_{f_i}^k\right) \qquad (3)$$

Corrector

Substitution of these expressions for the partial derivatives and simplification and Corrector can be written as follows:

$$\mathbf{U}_i^{**} = \mathbf{U}_i^k - \frac{\Delta t}{\Delta x}(\mathbf{F}_{i+1}^* - \mathbf{F}_i^*) - \mathbf{S}_i^* \Delta t$$

$$y_i^{**} = y_i^k - D^* \frac{\Delta t}{\Delta x} (V_{i+1}^* - V_i^*) - V \frac{\Delta t}{\Delta x} (y_{i+1}^* - y_i^*)$$
$$V_i^{**} = V_i^k - V^* \frac{\Delta t}{\Delta x} (V_{i+1}^* - V_i^*) - g \frac{\Delta t}{\Delta x} (y_{i+1}^* - y_i^*) - g \Delta t \left(S_o - S_{f_i}^*\right) \qquad (4)$$

Where ,

$$S_{f_i}^* = \frac{|V^*| V^* \cdot n^2}{D^{*4/3}}, \quad A^* = \left[\frac{B + (B + 2y_i^* * S)}{2}\right] * y^*, \quad D^* = \frac{A^*}{T^*}$$

Finally , interior vlue of v,y can be estimate by taking the average of predictor and corector as fllows :

$$\mathbf{U}_i^{k+1} = \frac{1}{2}(\mathbf{U}_i^* + \mathbf{U}_i^{**}) \qquad (5)$$

A SECOND NUMERICAL METHOD OF SOLUTION

As mention before lux schemes is used to verify McCormack sachem . it is a method of approximation . lux schemes has same boundary conditions which are the equation of positive and negative charstarstic line see figure 2 , whereas interior point of y,v can be written as (chaudhry),2003):

$$\begin{aligned} y_i^{k+1} &= \frac{1}{2}(y_{i-1}^k + y_{i+1}^k) - \frac{1}{2}\frac{\Delta t}{\Delta x} D_i^* (V_{i+1}^k - V_{i-1}^k) \\ &\quad - \frac{1}{2}\frac{\Delta t}{\Delta x} V_i^* (y_{i+1}^k - y_{i-1}^k) \\ V_i^{k+1} &= \frac{1}{2}(V_{i+1}^k + V_{i+1}^k) - \frac{1}{2}\frac{\Delta t}{\Delta x} g (y_{i+1}^k - y_{i-1}^k) \\ &\quad - \frac{1}{2}\frac{\Delta t}{\Delta x} V_i^* (V_{i+1}^k - V_{i-1}^k) + g \Delta t (S_o - S_f^*) \end{aligned} \qquad (6)$$

Where,

$$\frac{\partial f}{\partial x} = \frac{f_{i+1}^k - f_{i-1}^k}{2\Delta x}$$

$$\frac{\partial f}{\partial t} = \frac{f_i^{k+1} - f^*}{\Delta t}$$

$$f^* = \frac{1}{2}(f_{i-1}^k + f_{i+1}^k)$$

$$D^* = \frac{1}{2}(D_{i-1}^k + D_{i+1}^k)$$

$$S_f^* = \frac{1}{2}(S_{f_{i-1}}^k + S_{f_{i+1}}^k)$$

STABILITY AND ACCURACY

Stability of both schemes is controlled by Δt and Δx as follows:

- The positive characteristic through L has slope dt/dx = 1/(v+c).
- The negative characteristic through R has slope dt/dx = 1/(v-c) .

So, to keep within this domain, these criteria for the positive characteristic must be meeting: dt/dx > (v+c)

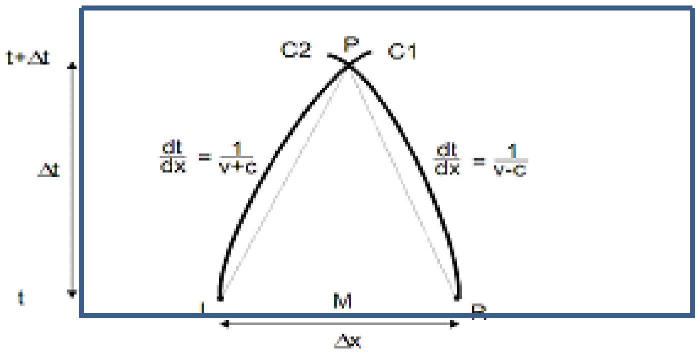


Figure 3: Characteristics around Point p

SIMULATION BY USING SPREADSHEETS

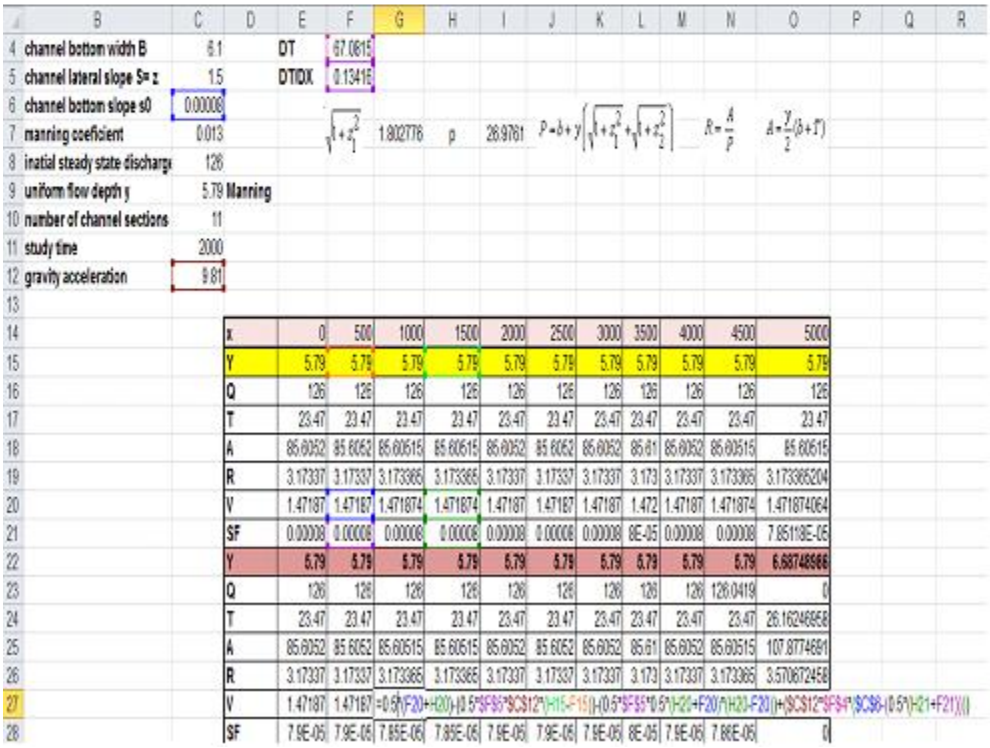
Solving the PDEs by using spreadsheet is one of the best technic. Graphical interface, numerical and visual result and fast calculating capabilities are one of the spreadsheets important advantages.

Changing any input will be festally reflected to the graph .FDM equation can be written easily then copied to other cells without writing the equations again to all cells separately which represent the iteration in spreadsheets .The objective of using spreadsheets in this paper is to use flexible tool in order to water flow Simulation.

The following input parameters are used for trapezoidal are : channel length from upstream to the dam, L = 5000 m; dam location, L = 5000m; initial water depth in channel y = 5.79m; acceleration due to gravity g=9.81 m2/s ; Manning roughness coefficient, n= 0.013, bed slope, So = 0.00008; and step size, dx = 500m; channel width, 6.1m ; channel lateral slope, z=1.5; initial steady state discharge, Q=126.

In order to obtain a stable solution for explicit simulation can be seen in cell F 4,5 in fig 4,5 where dt is equal to 67 according to the stability condition shown above .

The solution domain for the wave propagation due to sudden closure of the downstream gate based on lux scheme, which uses FDM, can be seen in Figure 4.



	B	C	D	E	F
4	channel bottom width B	6.1		DT	67.0815
5	channel lateral slope S= z	1.5		DT/DX	0.13416
6	channel bottom slope s0	0.00008			
7	manning coeficient	0.013			
8	inatial steady state discharge	126			
9	uniform flow depth y	5.79	Manning		
10	number of channel sections	11			
11	study time	2000			
12	gravity acceleration	9.81			

	D	E	F	G	H	I	J	K	L	M	N	O
14	x	0	500	1000	1500	2000	2500	3000	3500	4000	4500	5000
15	Y	5.79	5.79	5.79	5.79	5.79	5.79	5.79	5.79	5.79	5.79	5.79
16	Q	126	126	126	126	126	126	126	126	126	126	126
17	T	23.47	23.47	23.47	23.47	23.47	23.47	23.47	23.47	23.47	23.47	23.47
18	A	85.6052	85.6052	85.60615	85.60615	85.6052	85.6052	85.6052	85.61	85.6052	85.60515	85.60615
19	R	3.17337	3.17337	3.173365	3.173365	3.17337	3.17337	3.17337	3.173	3.17337	3.173365	3.173365204
20	V	1.47187	1.47187	1.471874	1.471874	1.47187	1.47187	1.47187	1.472	1.47187	1.471874	1.471874064
21	SF	0.00008	0.00008	0.00008	0.00008	0.00008	0.00008	0.00008	8E-05	0.00008	0.00008	7.85118E-05
22	Y	5.79	5.79	5.79	5.79	5.79	5.79	5.79	5.79	5.79	5.79	6.68748966
23	Q	126	126	126	126	126	126	126	126	126	126.0419	0
24	T	23.47	23.47	23.47	23.47	23.47	23.47	23.47	23.47	23.47	23.47	26.16246958
25	A	85.6052	85.6052	85.60615	85.60615	85.6052	85.6052	85.6052	85.61	85.6052	85.60515	107.8774691
26	R	3.17337	3.17337	3.173365	3.173365	3.17337	3.17337	3.17337	3.173	3.17337	3.173365	3.570672458
27	V	1.47187	1.47187	=0.5*(F20+H20)-(0.5*F5*C12*(H15-F15))-(0.5*F5*0.5*(H20+F20)*(H20-F20))+(C12*F4*(C6-(0.5*(H21+F21))))								
28	SF	7.9E-05	7.9E-05	7.85E-05	7.85E-05	7.9E-05	7.9E-05	7.9E-05	8E-05	7.9E-05	7.88E-05	0

Figure 4: Solution Domain of the Lux Scheme

Depending on our requirements we can easily generate FDM equations in the cell as much as need based on the Dx and Dt, which are the size of grids in the solution domain see attached spreadsheet.

For lux scheme, Solution of FDM equations is carried out based on Eq. (6) in the following spreadsheet format:

Left Boundary Condition Excel Code

(velocity)E27=F20-((SQRT(C12*F17/F18))*F15)+(C12*F4*(C6-F21))+((SQRT(C12*F17/F18))*E22)

Y= constant (5.79)

Right Boundary Condition Excel Code

V=0

(flow depth) O22 =(((SQRT(C12*N17/N18))*N15)+(C12*F4*(C6-N21))+N20)/(SQRT(C12*N17/N18))

Interior Value of V, Y Excel Code

(velocity)I27=0.5*(H20+J20)-(0.5*F5*C12*(J15-H15))-(0.5*F5*0.5*(J20+H20)*(J20-H20))+(C12*F4*(C6-(0.5*(J21+H21))))

(flow depth) =0.5*(H15+J15)-(0.5*F5*0.5*(H19+J19)*(J20-H20))-(0.5*F5*0.5*(J20+H20)*(J15-H15)).

For McCormack sachem, Solution of FDM equations is carried out based on Eq. (3,4,5) in the following spreadsheet format:

Left Boundary Condition Excel Code

(predictor velocity) E34=F24-((SQRT(C12*F19/F20))*F15)+(C12*F4*(C6-F26))+((SQRT(C12*F19/F20))*E27).

(corrector velocity) E35=E24-(F5*(E34)*(F34-E34))-(C12*F5*(F28-E28))-(C12*F4*(C6-E25))

Y= constant (5.79).

Right Boundary Conditions Excel Code

V=0

(predictor flow depth) O28 =O15-(F5*O21*(O24-N24))-(F5*O24*(O15-N15))

(corrector flow depth) O29 =(((SQRT(C12*N19/N20))*N15)+(C12*F4*(C6-N25))+N24)/(SQRT(C12*N19/N20)).

Interior Value of V, Y Excel Code

(predictor velocity) H34 =H24-(F5*(H24)*(H24-G24))-(C12*F5*(H15-G15))-(C12*F4*(C6-H26)).

(corrector velocity) H35 =H24-(F5*(H34)*(I34-H34))-(C12*F5*(I28-H28))-(C12*F4*(C6-H25))

(predictor flow depth) H28 =H15-(F5*H21*(H24-G24))-(F5*H24*(H15-G15)).

(corrector flow depth) H29 =H15-(F5*H30*(I34-H34))-(F5*H34*(I28-H28)).

Average velocity H26 =(H34+H35)/2

Average flow depth H27=(H28+H29)/2

	C	D	E	F	G	H	I	J	K	L	M	N	O
13													
14			0	500	1000	1500	2000	2500	3000	3500	4000	4500	5000
15		Y	5.79	5.79	5.79	5.79	5.79	5.79	5.79	5.79	5.79	5.79	5.79
16			5.79	5.79	5.79	5.79	5.79	5.79	5.79	5.79	5.79	5.79	5.79
17			5.79	5.79	5.79	5.79	5.79	5.79	5.79	5.79	5.79	5.79	5.79
18		Q	126	126	126	126	126	126	126	126	126	126	126
19		T	23.47	23.47	23.47	23.47	23.47	23.47	23.47	23.47	23.47	23.47	23.47
20		A	85.60515	85.60515	85.60515	85.60515	85.60515	85.60515	85.6052	85.61	85.6052	85.60515	85.60515
21		R	3.173365204	3.173365204	3.173365204	3.173365204	3.173365204	3.173365204	3.17337	3.173	3.17337	3.173365	3.17336520
24		V	1.471874064	1.471874064	1.471874064	1.471874064	1.471874064	1.471874064	1.47187	1.472	1.47187	1.471874	1.47187406
25		sf*	0.00008	0.00008	0.00008	0.00008	0.00008	0.00008	0.00008	8E-05	0.00008	0.00008	0.00008
26		SF	0.00008	0.00008	0.00008	0.00008	0.00008	0.00008	0.00008	8E-05	0.00008	0.00008	0.00008
27		Y	5.79	5.79	5.79	5.79	5.79	5.79	5.79	5.79	5.79	6.01471	6.68748986
28		y predector	5.79	5.79	5.79	5.79	5.79	5.79	5.79	5.79	5.79	5.79	6.68748986
29		y corector	5.79	5.79	5.79	5.79	5.79	5.79	5.79	5.79	5.79	6.23942	6.68748986
30		Q											
31		T	23.47	23.47	23.47	23.47	23.47	23.47	23.47	23.47	23.47	24.14413	26.1624695
32		A	85.60515	85.60515	85.60515	85.60515	85.60515	85.60515	85.6052	85.61	85.6052	90.95483	107.877469
33		R	3.173365204	3.173365204	3.173365204	3.173365204	3.173365204	3.173365204	3.17337	3.173	3.17337	3.273364	3.57067245
34		v predector	1.471874064	1.471874064	1.471874064	1.471874064	1.471874064	1.471874064	1.47187	1.472	1.47187	1.471874	0
35		v corector	1.471874064	1.471874064	1.471874064	1.471874064	1.471874064	1.471874064	1.47187	1.472	1.47187	0.581305	0
36		V	1.471874064	1.471874064	1.471874064	1.471874064	1.471874064	1.471874064	1.47187	1.472	1.47187	1.02659	0
37		sf*	7.85118E-05	7.85118E-05	7.85118E-05	7.85118E-05	7.85118E-05	7.85118E-05	7.9E-05	8E-05	7.9E-05	7.53E-05	0
38		SF	7.85118E-06	7.85118E-06	7.85118E-06	7.85118E-06	7.85118E-05	7.85118E-06	7.9E-06	8E-06	7.9E-05	3.68E-06	0

Figure 5: Solution Domain of the McCormack Scheme

One -dimensional representation of hydraulic heads can be seen in Figure 6 for McCormack sachem and Figure 7 for lux scheme. This figure shows the level of hydraulic heads over the solution domain. Each point of Figure 6,7 indicates the different levels of hydraulic heads. The flow head of the wave propagation can be seen in fig8 for 1050 second at which the water rises to reach the maximum flow head. Figure 8 shows the variation between flow depths for 1050 second by using McCormack sachem and lux scheme.

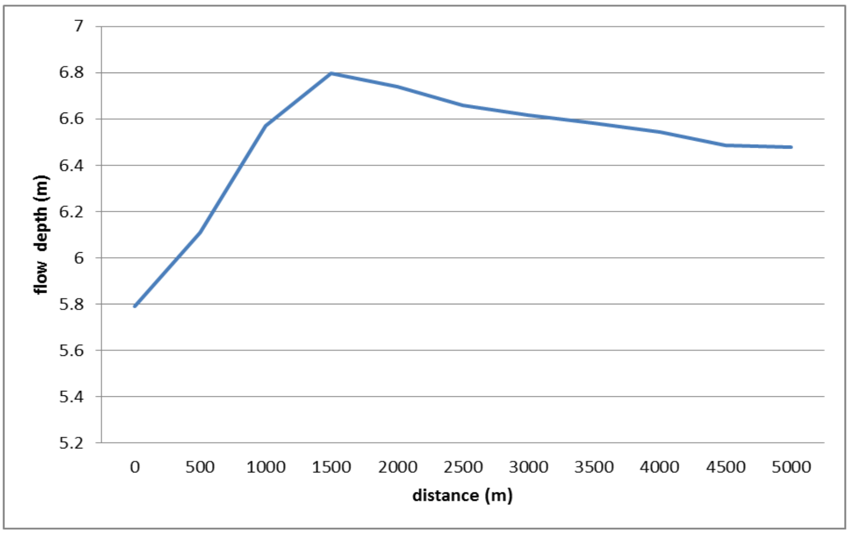


Figure 6: One-Dimensional Variation of Flow Depth Level (t=1050 Second) by Using McCormack Scheme

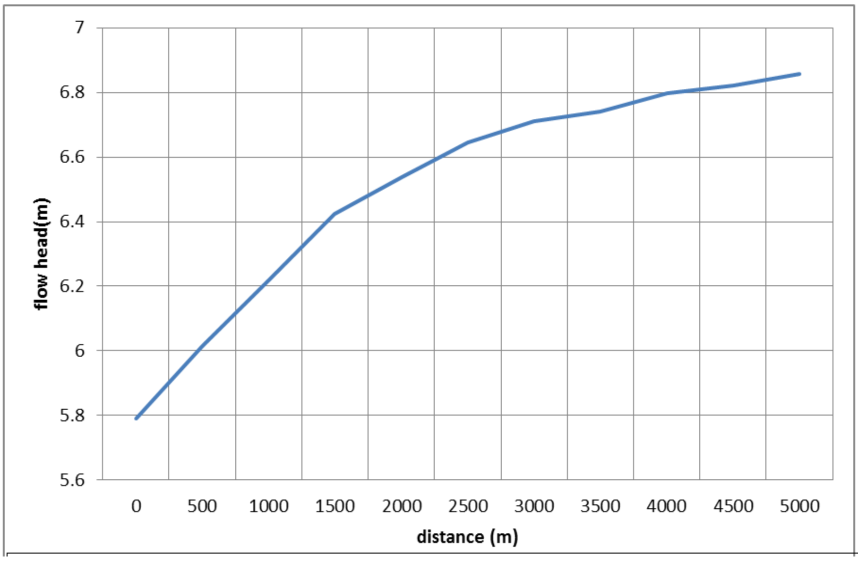


Figure 7: One-Dimensional Variation of Flow Depth Level (t=1050 Second) by Using Lux Scheme

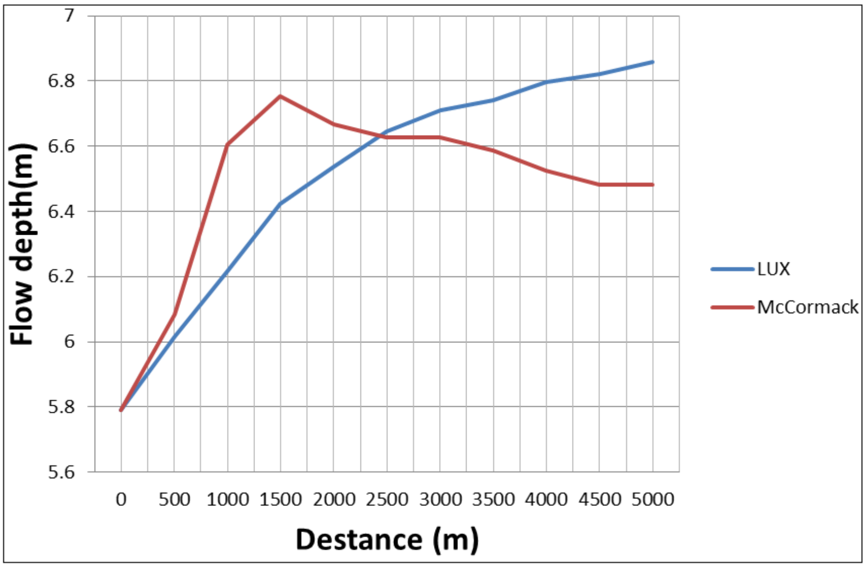


Figure 8: Variation between McCormack and Lux Scheme

CONCLUSIONS

Natural hazards occupy the essential and regional levels, hence, they are raised as a priority issues in road design and protection .The 2009 saudi Arabia floods affected Jeddah, on the red sea (western) coast. As of January 3rd, 2010, 122 people are reported to have been killed. Roads were under a meter of water. Unfortunately, Lack of knowledge in water flow modeling contributes to prevent manage flood risks. This paper provided the spreadsheet as solver used for solving finite deference. Finite deference of boundary conditions and interior value were written by using two different schemes (McCormack and lux scheme). The calculations have been done iteratively. One of the main purposes of this paper is to try to match or meet the scientific methodology for studying such phenomena through using one of numerical solution scheme and anther for verification. The variation between tow schemes is quite large. Lux scheme has been compared with Hanif Chaudhry results and found that the result is same. Figures show lux scheme is more logical than McCormack scheme.

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